Computer aided design and pattering of tensioned fabric structures

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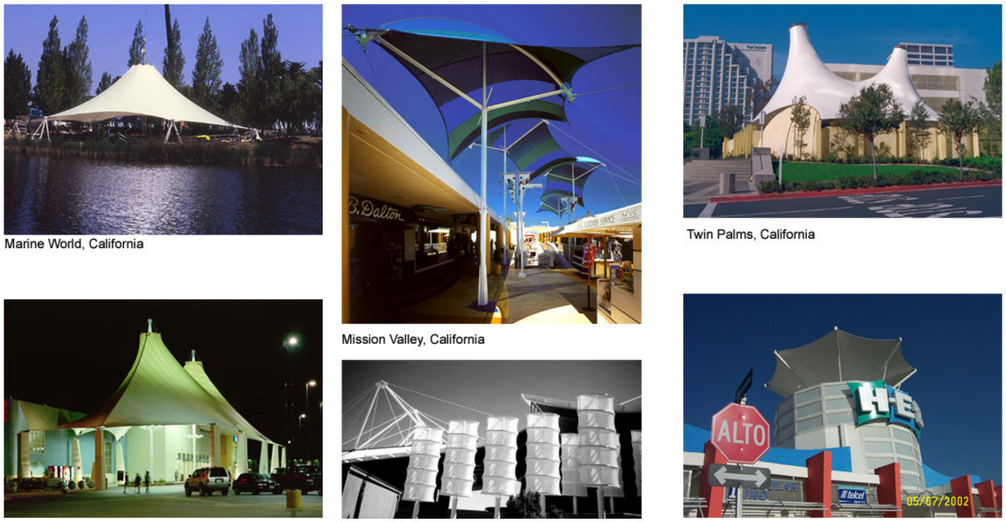
Abstract

This paper presents an overview for a Computer-Aided Design and Patterning of Tensioned Fabric Structures. After a brief introduction to tensioned structures and their applications in building, a description of tension fabric structure design is presented. A computer program called Mathform, which is used for analysis design and patterning of tensioned fabric structures utilizing the Dynamic Relaxation method, is also discussed. Finally two design examples created with the Mathform program are presented.

Introduction [ASI 00]

Tension fabric structures are one of the most exciting and rapidly developing technologies in the building industry today. Material advances in coated woven textiles combined with design and numerical techniques for developing membrane structures have yielded a new building form for permanent architectural applications (see figure 1). Among them are complete architectural fabric enclosures for buildings, airport terminals, restaurants, and other public spaces: large span structures such as stadiums, arena enclosures and retractable membrane systems for covering indoor and outdoor spaces.



Marine World, California

Mission Valley, California

Twin Palms, California

HEB, Texas

Santa Fe Opera House, New Mexico

HEB, Mexico

Figure 1- Examples of Membrane Forms

Perhaps the most exciting aspects of fabric structures are the remarkable variety of anticlastic forms that can be realized. These include hyperbolic shapes, saddles, cones, domes, vaults, and waved and plate types. The choices are endless. The range of forms is augmented through the use of support and restraint elements such as cables, masts, trusses, and rigid nodes. Cable-membrane structures are referred to as “form active systems” since the form being derived from the direct relationship between force and cable structures. This concept may be referred to as **"form follows force".**

Design of Tension Fabric Structures

The design of tension fabric structures begins with a form conceived by the designer. A drawing or a physical model usually represents this form. The desiger's form provides basic concept and support conditions which allow an engineer to find the true shape of the structure. (It is interesting to note the concept of form finding and not form giving).

The engineer usually employees the following approaches for form finding:

Qualitatively - through physical modeling historically used by designers like Frei Otto to analyze and design membrane structures. Physical modeling involves creating a scaled model made of materials, which depict the actual structure (e.g. textile cloth for the fabric, wire for edge cables etc.). If the structural properties of the material of the model are known this model can also be a structural testing model for load analysis. The great advantage of the physical modeling method is the explanation of the physical behavior of the actual structure.

Quantitatively[CAP97] **-** using mathematical tools. Mathematics does not explain physical behavior; it only describes it [SAL75]. However, in recent years, with the help of powerful computers, engineers can easily solve nonlinear equations and track out complex trajectories that cannot be drawn. Mathematical descriptions are now so efficient that powerful computers can easily and fully conceive and explain membrane structure behavior. Computer simulation of the structure has become a valuable tool to help the designer find realistic shapes.

The design of membrane structures regardless of the methods used, has three steps:

1) Form-finding or Initial geometry formulation
2) Engineering analysis and membrane design
3) Patterning

Computational methods

Tension fabric structures can be designed either by using physical modeling or computer methods. Due to the variety of alternate design solutions to a fabric problem that can be quickly achieved utilizing computers, computer methods would be the favored tool of engineers for the design of fabric structures. A number of computer methods have been developed for analysis of geometrically non-linear fabric structures, which include shape (form-finding). The following is a brief description of a computer program, Mathform. The program code is being written in Visual Basic and embedded into AutoCAD. It uses Microsoft Access as the backend storage. The minimum hardware requirement for running Mathform is a Pentium processor PC (or higher) with a Windows operating system. Note that the program can be utilized for analysis of space truss structures, however, the emphasis here is on the application of the program for the analysis and design of fabric structures.

Mathform Program Description

The analysis portion of the Mathform program is based on the published works of Dr. Michael Barnes [BAR77] [BAR86] [BAR84] and the work of Dr. Tajav Deganayar [DEG], who was one of the first to implement the dynamic relaxation method in the United States to develop a software program called SOFTSPACE (written in FORTRAN). This program has been adopted by designers and engineers at Advanced Structures Inc. to design fabric membrane structures.

Mathform is a nonlinear analysis and patterning program. The analysis section of the program is based on the dynamic relaxation method. The basis of the method is a step-by-step algorithm tracing the motion of a structure until the structure reaches equilibrium due to damping. The dynamic relaxation methods solve the geometric nonlinear problem of form finding by equating it to a dynamic problem. The dynamic problem is then solved using the principle of dynamics. It is suitable for computer simulations of tensile structures. The method can easily take into account nonlinear behavior resulting from large deformations.

The analysis of fabric structure by Mathform utilizing Dynamic Relaxatation with kinetic damping involves the following steps:

1- Establish a coordinate system with nodal point coordinates and consider a line with two connecting nodes I and K in an arbitrary location in space, as a three dimensional linear elastic truss element. Let XYZ form the global coordinate system as shown in figure 2.

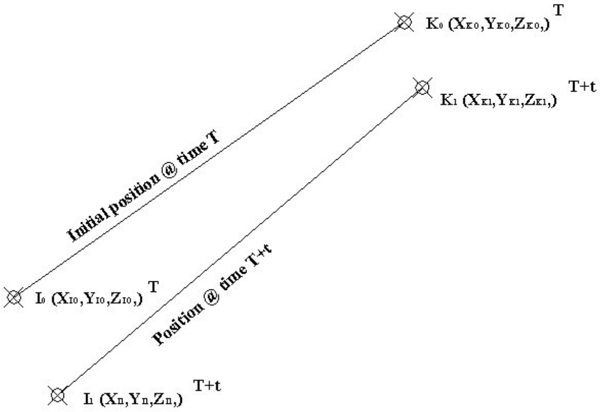


FIGURE 2

2- Establish element types to be used for modeling various elements.

a) Constant tension fabric element (usually used during form finding).
b) Constant force density fabric element (usually used during form finding).
c) Tension only element (capable of taking only tension).
d) Three dimensional truss element (capable of taking both tension and compression).
e) Slack element (zero stiffness element).

3- Generation of element properties, connectivity.

a) Pretension (T).
b) Modulous of elasticity (E).
c) Element length from node connectivity (RL).

4- Define support boundary condition.

5- Consider a simple network of four cables as shown in figure-3.

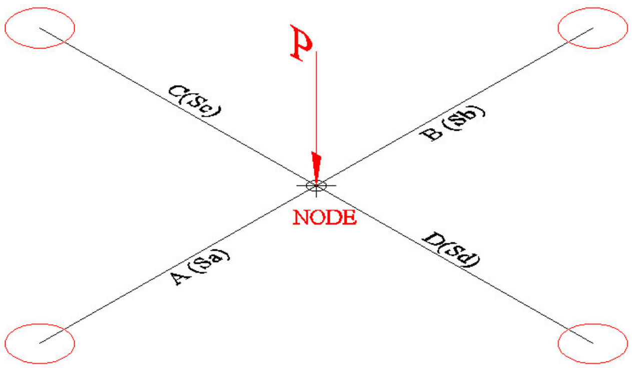


FIGURE 3

$$R' := \sum_{n} F' - P \tag{1}$$

R'	Residual force at node i in the direction being considered.
$\sum_{n} F'$	Sum of the internal member forces in the direction being considered.
P	Applied load at node in the direction being considered.

Any residual force will be attributed to the dynamic behaviour at the node.

R^{T}	From Newton's second law	$R^{T} := M(i) \cdot a$	(2)
V^{T+t}	From Newton's equation of motion	$V^{T+t} := V^{T} + a \cdot t.$	(3)
$M(i)$	Mass at node i		
a	Acceleration of the node in the time interval t.		
V^{T}	Initial velocity of the node at the start of time interval t.		
V^{T+t}	Final velocity of the node at the end of time interval t.		

6- Generate mass matrix for each element.

$$M := \frac{(E \cdot A + T(R))}{RL} \tag{4}$$

7- Compute mass matrix for each node by summing of the mass of individual member coming to the node.

$$M(i) := \sum_{n} M \tag{5}$$

8- Compute the velocity at time T+t.

$$V^{T+t} := V^{T} + \frac{R^{T}}{M(i)} \cdot t \tag{6}$$

9- Compute displacement D at the end of time interval t.

$$\Delta := t \cdot V^{T+t} \tag{7}$$

10- Update X^{T+t}, Y^{T+t} & Z^{T+t} coordinates of node i at the end of time interval t.

$$X^{(T+t)} := X^{T} + \Delta \tag{8}$$

11- Compute change in coordinated Dx, Dy & Dz

$$Dx^{T+t} := X^{T+t} - X^{T} \tag{9}$$

12- Compute new tensions at the end of time interval t.

$$T^{T+t} := T + E \cdot A \cdot \left(\frac{RL^{T+t} - RL^{T}}{RL^{T}} \right) \tag{10}$$

13- Update residuals at node i at the end of time interval t.

$$R^{(T+t)} := P_{xi} + \left(\frac{Dx}{RL}\right)^{(T+t)} \cdot T^{T+t} \qquad (11)$$

14- Check for local peak in kinetic energy.

Compute old & new kinetic energy using velocities V^T & V^{T+t}

$$KE_{old} := \left(\frac{1}{2}\right) M \cdot \left(V^T\right)^2 \qquad (12) \qquad\qquad KE_{new} := \left(\frac{1}{2}\right) M \cdot \left(V^{T+t}\right)^2 \qquad (13)$$

15- If KE_{new} is found to be less than KE_{old} than the peak has been passed and velocities are set to zero (Figure 4 depicts a typical kinetic energy peaks & reset).

16- The first velocities on restarting the process (by assuming the peak to occurs at mid point of the first time step) are given by:

$$V^{\frac{t}{2}} := \left(\frac{t}{2 \cdot M(i)}\right) \cdot R^T \qquad (14)$$

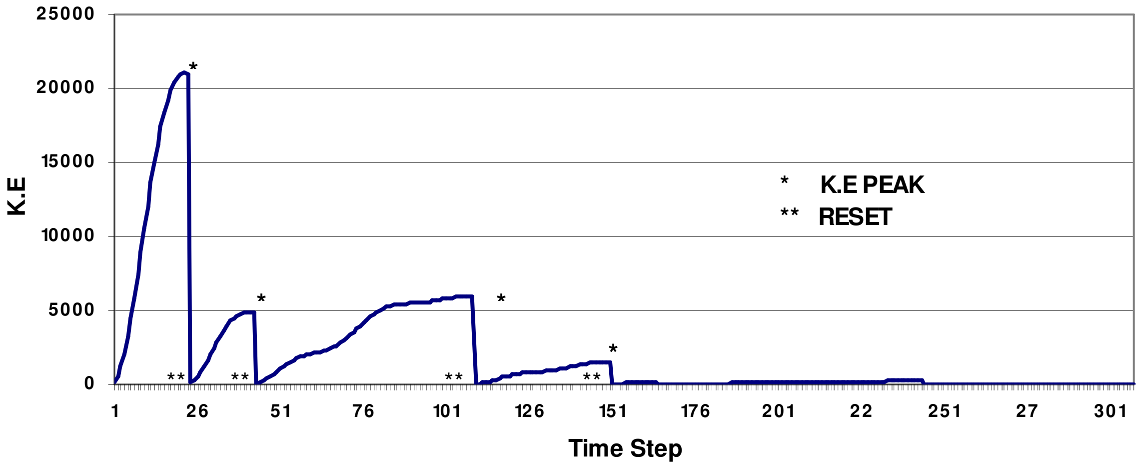


Figure 4 Kinetic Energy Plot

17-Repeat step 8 to update velocities until the next energy peak. Iterate until the solution is arrived at when motion of a node comes to rest due to damping (i.e. the residual forces are sufficiently small) and update the final geometry.

18- Using the new geometry, apply loads and desired load combinations.

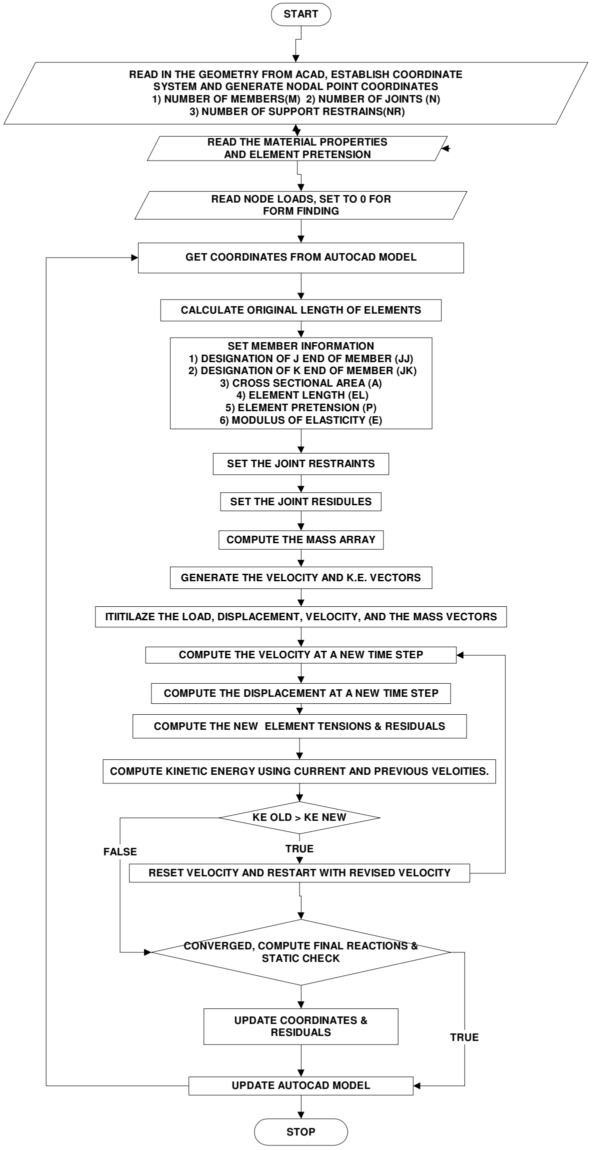
19- Solve system of equations by iteration and find nodal displacement and member forces.

20- Review the results and, if necessary, reanalyze the modifying input parameters.

Application of Mathform for form finding:

In the Mathform form-finding process the form could be found either by specifying a constant tension or by specifying a force density ratio (given by tension/length) during step-12 while updating tensions. Specifying constant tension would give minimal surface and using force density would form a curvature in form finding.

The following is a flowchart for the program.



Patterning

Fabric structures usually cover a three-dimensional space even though the membrane cover by itself is a two dimensional surface. Patterning is a process of mapping a curvilinear surface to a flat surface or, mathematically speaking, it is the process of transforming a two-dimensional surface in a three-dimensional coordinate system into a two-dimensional coordinate system with geometrical conformity.

The patterning of fabric structures using Mathform involves the following steps:

1) Triangulate the model using Delauney's [HAN86] triangulation and using the coordinates of the triangles to draw 3dface in the AutoCAD model.
2) Flatten of selected strips from the AutoCAD 3Dface model by transforming 3Dfaces in X, Y, Z coordinates to 3Dfaces in X, Y plane coordinate.
3) Calculate flattened strip (free of pretension) by applying appropriate compensation factor in the warp and fill direction of the fabric strip
4) Create compensated strip drawings for fabrication.

Design of Membrane Structures using Mathform

Mathform is used for three purposes: a) As a form finding tool b) As an engineering tool and c) As a patterning tool.

The following is a brief explanation of how to execute Mathform.

In order to run Mathform, the fabric model is generated in Auto CAD. The process of generating a three dimensional tension membrane model involves creating a flat finite element fabric net model, with properties in warp and fill direction that correspond to the behavior of the real fabric. The fabric net mesh is structured and uniform. Edge cables, masts, webbing or fabric reinforcement, having different properties, are modeled as a sequence of different line elements. The support boundary conditions are applied by restraining the selected nodes (figures 5 & 7). The attributes for elements like the material property and pretension are specified. The form finding process, which is either minimal surface or force density method selection, completes the model generation process for form-finding.

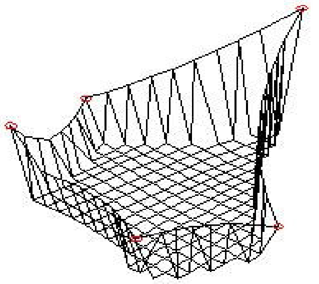


FIGURE 5

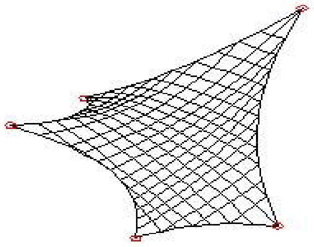


FIGURE 6

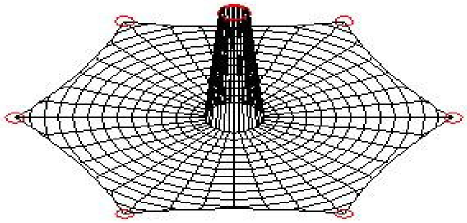


FIGURE 7

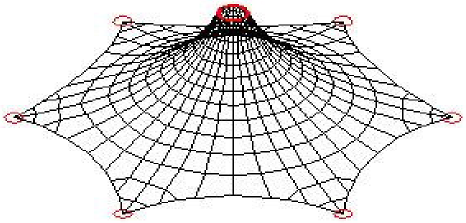


FIGURE 8

Form-finding

Form-finding, or initial geometry formulation, provides a detailed geometric description of the structure. In this phase, the shape of the structure is determined by assigning the proper prestress forces to the fabric network and specifying the support boundary condition such as: masts, arches, perimeter beams, etc. The initial shape of the fabric is approximated (figures 5 & 7) and then the pretension analysis establishes the final shape of the structure (figures 6 & 8).

Engineering analysis and membrane design

After the final shape of the structure is computed through form finding, the structure is loaded for different load cases. Due to the lightweight of the structure, the dead and seismic loads are neglected and wind or snow loads usually govern the design. Due to the nonlinear nature of the problem, the principle of superposition does not hold. Hence the load analysis should be carried out independently for each load case. The computed forces in the fabric can be used to compare against the allowable values. Critical reactions for the load case in the computer runs are used for the design of the supporting structure. The maximum displacement of the structure is used to compare against the allowable deflection per the appropriate code.

Patterning

The form finding model is triangulated and 3Dfaces are drawn on top of form-found model as shown in the figure 9. This process is called 3D facing. After 3Dfaceing is complete, strips are selected for flattening. The following criteria are considered during strip selection [HAN86]:

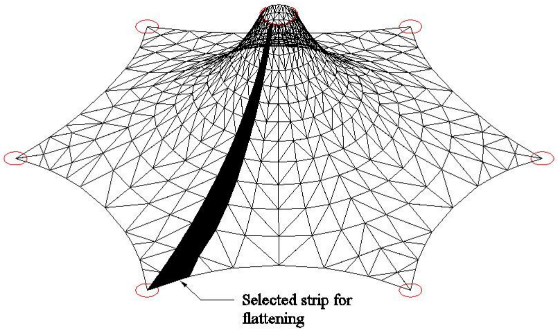


FIGURE 9

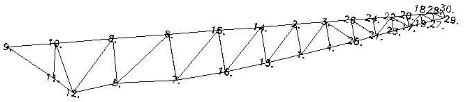


FIGURE 10: Selected strip from 3dface model

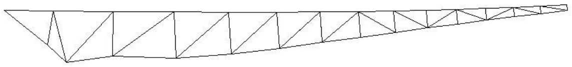


FIGURE 11: Flattened strip (x & y coordinates only)

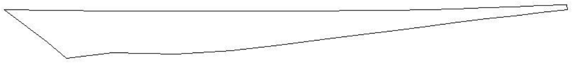


FIGURE 12: Compensated strip for fabrication

- Visual and architectural effects usually determine the orientation of the strips (seam alignment).
- Parallel orientation of these strips is suitable for saddle-shaped (Hyper like) surface.
- Radial orientation of the strips must be applied for radial geometry structures.
- Orientation of each strip has to be fixed in such a direction that the warp and fill follow the direction used in the form finding model.
- The width of the membrane strips depends on the manufactured fabric width and on minimizing the waste of material.

After flattening the stress-free lengths (slack length) of all cables, membrane pieces are determined by applying the required compensation refer to figures 10, 11 & 12.

Conclusion

Tension fabric structures are a developing technology, which gives architects and engineers the ability to experiment with forms and create exciting solutions to conventional design problems. Tension fabric structures can be designed either by using physical modeling or computer methods. The possibility of realizing a diverse range of forms in a short period of time makes computer simulation the preferred method of designing tension fabric structures in the future. Mathform is one of the few fabric structure analysis software programs in the United States today. Its fast solver, friendly interface and integration within AutoCAD allow engineers to come up with solutions to numerous structural fabric applications, from the simple to the most complex.

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